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| **Lesson Title:**  **FACs, FIGs, & FORMs:**  **The Real Facts about Geometric Figures and their Measurement Formulas** |
| **Subject area / course / grade level:**  **Geometry & Measurement / Math / Grades 4 – 6** |
| **Introduction:**  This lesson is designed to focus on a major content topic in the measurement curriculum – understanding how surface area and volume formulas are derived for common geometric figures, including both 2-D shapes and 3-D solids. Students encounter these shapes, especially solids, early on and study these shapes throughout their school grades. In particular, elementary students are frequently introduced to common measurement formulas by mere presentation and often have to resort to memorizing them without fully understanding the general patterns supporting the formulas’ development. Memorization alone rarely allows for easy use or application of the measurement formulas within varied problem solving contexts. Because students’ constructive knowledge of how these formulas are derived is limited, their ability to apply and/or recall the formulas may be limited as well. |
| **Lesson Length:**  **90 – 120 min (1.5 – 2 hrs)** |
| **Materials:**  Square dot-grid paper and/or geoboard w/ bands  Handout to record data and derived formulas  Handout of Practice Exercises  Debrief/Reflection Handout  Sets of clear geo-solids (per group)  Circular cut-outs and scissors (*or* Sets of Fraction Circles – commercial manipulative)  Various circular objects (e.g., paper plate, round trash can, water bottle)  Tape Measures (flexible type)  Calculators |
| **Lesson Overview:**  Participants will use inductive reasoning (via working through multiple examples) to derive the perimeter and area formulas for common geometric shapes (including the rectangle, square, parallelogram, triangle, trapezoid, and circle as well as solids like prisms and pyramids). There are two major parts to the Lesson: Part 1 - *Deriving Perimeter and Area formulas for common 2-D shapes* and Part 2 - *Deriving (Surface) Area formulas for common 3-D shapes (Prisms and Pyramids).* |
| **Tennessee Standards:**  GLE 0406.4.2 – Understand and use measures of length, area, capacity, and weight.  GLE 0406.4.3 – Solve problems that involve estimating and measuring length, area, capacity and weight.  GLE 0506.4.1 – Use basic formulas and visualization to find the area of geometric figures.  GLE 0506.4.2 – Describe polyhedral solids and analyze their properties, including volume and surface area.  GLE 0506.5.1 – Make record, display and interpret data and graphs that include whole numbers, decimals, and  fractions.  GLE 0606.4.3 – Develop and use formulas to determine the circumference and area of circles, and the area of  trapezoids, and develop strategies to find the area of composite shapes.  GLE 0606.4.4 – Develop and use formulas for surface area and volume of 3-dimensional figures.  GLE 0606.1.8 – Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to  facilitate problem solving, and to create accurate and reliable models of mathematical concepts.  **MCS/Math Power Standards:**  Grade 4 - Understand and use measures of length, area, capacity and weight.  Grade 5 - Find the area, volume and surface area of geometric figures including irregular shapes, prisms and  polyhedral solids.  Grade 6 - Develop and use formulas to determine the circumference and areas of circles and the area of  trapezoids and develop strategies to find the area of composite shapes.   * Determine surface area and volume of prisms, pyramids and cylinders. |
| **Lesson objective/outcome(s):**  **TLW:** Develop and apply content knowledge of how perimeter formulas are derived for common 2-D shapes.  Develop and apply content knowledge of how (surface) area formulas are derived for common 2-D  (and 3-D) shapes. |
| **ENGAGE**   * How will the teacher capture students’ interest? * What kind of questions should the students ask themselves after the engagement?   **Activity:**  Facilitator will pose this scavenger hunt/scenario to the group:  *“You have 1-2 minutes to gather several items from your purse, briefcase, (etc.) which show examples of polygons and/or solids. Briefly share your findings with a neighbor. Select one item to tell how you would determine the measurement attribute (perimeter, circumference, and/or surface area) of the item. Each person should have at least one turn.”*    **Questions:**  1. What observations did you make as you gathered items?  2. What terms/vocabulary did you use to describe the measurement?  3. To what extent did you incorporate measurement formulas?  4. How do similar measurements of different items compare to each other? |
| **EXPLORATION**   * What hands-on/minds-on activities will students be doing? * What are the “big idea” conceptual questions that the teacher will use to encourage and/or focus students’ exploration?   **PART 1: Deriving Perimeter and Area formulas for common 2-D shapes**   * As per formula for perimeter, make the square and rectangle on the geoboard; observe perimeter measurement patterns and derive the formulas based on pattern observations (**P=4*s*** for square; **P=2*l* + 2*w*** for rectangle). * As per formula for the circle, measure the **distance around (circumference)** and the **distance across (diameter**) several circular objects in the room. Measure as precisely as possible in the smallest unit possible (e.g., mm). Calculate the ratio of the circumference to the diameter to get**, = (≈3.14159…).** To account for error and get closer to the constant, take the average of several ratios. This yields ***C* =**  for the circle. * Record all necessary data on the handout to use later in the lesson. * As per formula for area, make each polygonal shape on the geoboard one at a time. To see how the area formulas build on each other, consider making and discussing each shape in this order: square (**A=*s2***), rectangle (**A=*lw***), parallelogram (**A=*bh***), triangle (**A= ½*bh***), and trapezoid (**A= ½*(b1+b2)h***). Observe area measurement patterns; derive the formulas based on pattern observations. Record all data on the handout. * As per formula for area of the circle, use fraction circles (or circular cut-outs); take the “sector” pieces from one circle and arrange them in a parallelogram. Determine the area of the parallelogram using the formula above; the base of the parallelogram is half the circumference of the circle and its height is the radius of the circle. Thus, the area of the circle = the area of the parallelogram = **½**(***C***)(***r)*** = **½(2*r*)(*r)*** = ***r*2.** * Work through practice exercises to apply the formulas to basic geometric figures, including irregular shapes. * Debrief and reflect on the math content and processes used in PART 1 Lesson.   **PART 2: Deriving (Surface) Area formulas for common 3-D shapes (Prisms and Pyramids)**   * As per surface area formula for the prism, begin by examining the “nets” for different type prisms; observe area patterns and derive the formulas based on pattern observations. For any general right prism, **S.A. = 2*B* + *ph,*** where ***B***=area of base; ***p***=perimeter of base; and ***h***=height of prism. * Apply or transfer the prism formula to the cylinder’s surface area and note similarities/differences. * Record all necessary data on the handout. * As per surface area formula for the pyramid, begin by examining the “nets” for different type pyramids; observe area patterns and derive the formulas based on pattern observations. For any general right pyramid,   **S.A. = *B* + *pl,*** where ***B***=area of base; ***p***=perimeter of base; and ***l***= (slant) height of each triangular face (not the height of the pyramid itself).   * Apply or transfer the pyramid’s formula to the cone’s surface area and note similarities/differences. * Record all necessary data on the handout. * Work through practice exercises to apply the formulas to basic geometric figures, including irregular shapes. * Debrief and reflect on the math content and processes used in PART 2 Lesson.   **Hands-on/Minds-on Activities (refer also to PART 1 and PART 2 above):**  1. Making different geometric (2-D) shapes on the geoboard; determining measurement attributes of each.  2. Measuring circular objects; determining measurement attributes of each.  3. Data collection of measurement attributes (e.g., length, width) for different shapes.  4. Determining patterns that appear in the data from the entire group.  5. Drawing conjectures about the observed patterns in the data that relate to the measurement formulas.  **Big-Idea Questions include:**  1. Where do the perimeter & area formulas come from?  2. How do the formulas of different shapes relate to each other?  **At-Home Exploration:**  Take time to explore the following websites. Feel free to add to the list.  [www.virtualmanipulatives.com](http://www.virtualmanipulatives.com)  [www.mathplayground.com](http://www.mathplayground.com)  [www.jmathpage.com](http://www.jmathpage.com)  [www.ixl.com/math](http://www.ixl.com/math) |
| **EXPLANATION**   * Student explanations should precede introduction of terms or explanations by the teacher. What questions or techniques will the teacher use to help students connect their exploration to the concept under examination? * List higher order thinking questions which teachers will use to solicit *student* explanations and help them to justify their explanations.   **Questions:**  1. Why are these formulas significant to know?  2. When/where can these formulas be applied in real-life settings?  3. List real-life examples in which perimeter formulas may be applied.  4. List real-life examples in which area formulas may be applied.  5. List real-life examples in which surface area formulas may be applied  6. How might you/family use these formulas on a daily basis? |
| **ELABORATION**   * Describe how students will develop a more sophisticated understanding of the concept. * What vocabulary will be introduced and how will it connect to students’ observations? * How is this knowledge applied in our daily lives? * Have participants choose an object at home/in classroom which he/she will apply one of the formulas. * Reflect on all the different aspects of the formula and explain your findings to a neighbor/friend.   **Vocabulary:**  area, base, circle, circumference, data, formula, height, length, measurement, parallelogram, pattern, plane figure, polygon, prism, pyramid, radius, two-dimensional (2-D), rectangle, solids, square, surface area, three-dimensional (3-D), trapezoid, triangle, vertices |
| **EVALUATION**   * How will students demonstrate that they have achieved the lesson objective? * This should be embedded throughout the lesson as well as at the end of the lesson * Handout of practice exercises * Debrief/Reflection on all tasks * On-going discussions |

**NOTES:**